such a lightweight structural component. As more superplastic alloys such as 7475 aluminum become commercially available, this type of investigation will undoubtedly attract more attention from the researchers in the area of lightweight structures.

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Radiative Effect on the Conjugated Forced Convection-Conduction Analysis of Heat Transfer in a Plate Fin

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Nomenclature

C_p = specific heat at constant pressure
C_T = temperature difference parameter, $T_\infty/(T_0-T_\infty)$
f = reduced stream function
$h^*, \hat{h}^*$ = dimensional and dimensionless modified heat-transfer coefficients
k, k_f = fluid and fin thermal conductivity
L = fin length
N_c = convection-conduction parameter, $(kL/k_f\delta)Re_L^{1/2}$
N = conduction-radiation parameter, $k\beta^*/4\sigma(T_0-T_\infty)^3$
Pr = Prandtl number, ν/α
q^r = radiative heat flux, $(-4\sigma/3\beta^*)(\partial T^4/\partial y)$, for optically thick limit approximation
Q = overall heat-transfer rate
T, T_f = fluid and fin temperature
T_0 = root temperature
u, v = velocity components in x and y directions, respectively
x, y = coordinates system shown in Fig. 1
α = thermal diffusivity
β^* = extinction coefficient
δ = half-thickness of the fin
ξ, η = pseudosimilarity variables
θ, θ_f = dimensionless fluid and fin temperatures
ν = kinematic viscosity
ρ = density of fluid
σ = Stefan-Boltzmann constant
ψ = stream function

Subscripts

w = condition at wall
∞ = condition at freestream

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Introduction

In the conventional heat transfer analysis of fins, it is standard practice to assume that the heat transfer coefficient for convection at the fin surfaces is uniform. However, there is evidence in the literature demonstrating that the heat-transfer coefficient can experience substantial variations along the fin surfaces.[1,2]

This Note is concerned with fins that transfer heat to a surrounding fluid by forced convection and radiation. The heat-transfer coefficient along the fin is not prescribed, but rather solved in advance from the boundary-layer convection flow. Therefore, the modified local heat transfer coefficient is determined by a highly coupled interaction among the fin conduction, radiation, and fluid convection flow.

The problem to be analyzed here is illustrated in Fig. 1. A vertical fin of length L and thickness 2δ is extended from a wall at temperature T_0 and situated in a uniform freestream having temperature T_∞ and velocity u_∞. The optically thicken limit approximation for the radiative flux is assumed, and the tip for the fin is insulated.

Governing Equations

Consider now a vertical fin parallel to a uniform freestream. Let x and y denote, respectively, the streamwise and normal coordinates. The conservation equations for a laminar boundary layer over a vertical fin is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q^r}{\partial y} \tag{3}$$

subjected to the following boundary conditions:

$$u=v=0,\quad T=T_w,\quad \text{at } y=0$$

$$u=u_\infty,\quad T=T_\infty,\quad \text{as } y\to\infty \tag{4}$$

Introducing pseudosimilarity variables (ξ,η) with a reduced stream function $f(\eta)$ and a dimensionless temperature $\theta(\xi,\eta)$ as

$$\xi=\frac{x}{L},\quad \eta=\left(\frac{y}{L}\right)Re_L^{1/2}/\xi^{1/2},\qquad f(\eta)=\psi\frac{(x,y)}{(u_\infty L\xi\nu)^{1/2}},$$

$$\theta=\frac{T-T_\infty}{T_0-T_\infty} \tag{5}$$

where the stream function ψ satisfies the continuity equation with $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$.

When the transformation is applied to Eqs. (2-4) and the optically thick limit approximation for q^r is assumed, the governing system becomes

$$f''' + \tfrac{1}{2}ff'' = 0 \tag{6}$$

$$Pr^{-1}\theta'' + \frac{1}{2}f\theta' + \frac{4[(\theta+C_T)^3\theta']'}{3PrN} = \xi f'\frac{\partial\theta}{\partial\xi} \tag{7}$$

$$f=f'=0,\ \theta_w=[T_w(x)-T_\infty]/(T_0-T_\infty),\ \text{at } \eta=0$$

$$f'=1,\ \theta=0,\ \text{as } \eta\to\infty \tag{8}$$

where the primes denote partial differentiation with respect to η.

Considering a very thin fin, it is reasonable to assume a one-dimensional model for the fin temperature along the x-coordinate. The fin conservation energy equation is

$$\frac{d^2T_f}{dx^2} = \frac{h^*(x)}{k_f\delta}(T_f-T_\infty) \tag{9}$$

The associated boundary conditions are

$$T_f=T_0,\ \text{at } x=L \text{ and } \frac{dT_f}{dx}=0,\ \text{at } x=0 \tag{10}$$

The basic coupling between the fin and the convective flow is expressed by the requirement that the fin and fluid temperatures and heat fluxes be continuous at the fin/fluid interface at all x positions.

$$T_f=T_w \text{ and } -k\frac{\partial T}{\partial y}+q^r=h^*(T_f-T_\infty),\ \text{at } y=0 \tag{11}$$

Equation (9) was recast in dimensionless form by the substitution

$$\xi=x/L,\ \theta_f=(T_f-T_\infty)/(T_0-T_\infty) \tag{12}$$

Equation (9) then becomes

$$\frac{d^2\theta_f}{d\xi^2} = N_c\hat{h}^*\theta_f \tag{13}$$

where N_c is the convection-conduction parameter $[N_c=(kL/k_f\delta)Re_L^{1/2}]$ and $\hat{h}^*$ is a dimensionless local heat transfer coefficient with radiative effect for the current cycle of the iteration.

$$\hat{h}^* = -\left(1+\frac{4(\theta+C_T)^3}{3N}\right)\frac{\partial\theta}{\partial\eta}\Big/(\theta_f\xi^{1/2}),\ \text{at } \eta=0 \tag{14}$$

The method to be used here involves a succession of consecutive solutions for the fin and boundary layer. Initially, the boundary-layer equations are solved subject to an assumed fin temperature distribution that satisfies the thermal boundary conditions, and from this solution $\hat{h}^*$ is evaluated. By substituting $\hat{h}^*$ into the fin energy equation, we get a new value for θ_f. This new θ_f is imposed as the fin surface boundary condition for the governing equations. This procedure of alternately solving the boundary-layer problem and fin conduction problem was continued until convergence was attained.

Numerical Procedure

The solutions of the boundary-layer equation were obtained by an implicit finite-difference method devised by Cebeci and Bradshaw.[3] According to this method, Eqs. (6) and (7) are first written in the first-order equation by introducing new known functions. The functions and their derivatives in the first-order equations are then approximated by centered difference quotients and averaged at the midpoints of net rectangles to yield finite difference equations. The resulting nonlinear difference equations are finally solved using Newton's method. The fin conduction equation is also written in the finite difference form. For small ξ, a finer ξ subdivision was needed for the boundary-layer and heat conduction equations. The present method uses 45 points in the streamwise direction and 61 grid points in the cross-stream direction.

Results and Discussion

Numerical results of the overall rate of heat transfer Q from the fin can be obtained from the heat conducted from the wall into the fin base at $\xi=1$ or by integrating the local heat flux at the fin surface. The corresponding heat flux values of these two methods are found to be in agreement. They may be expressed in dimensionless form as follows in Eq. (15)

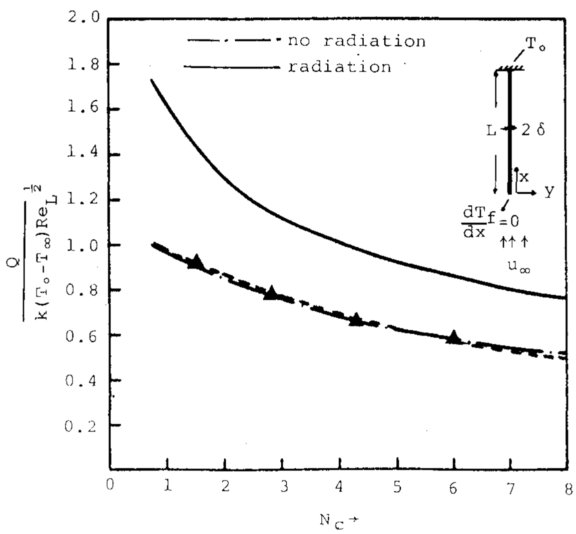

Fig. 1 Total heat-transfer rate. ---, simple model; ▲, Sparrow.[4]

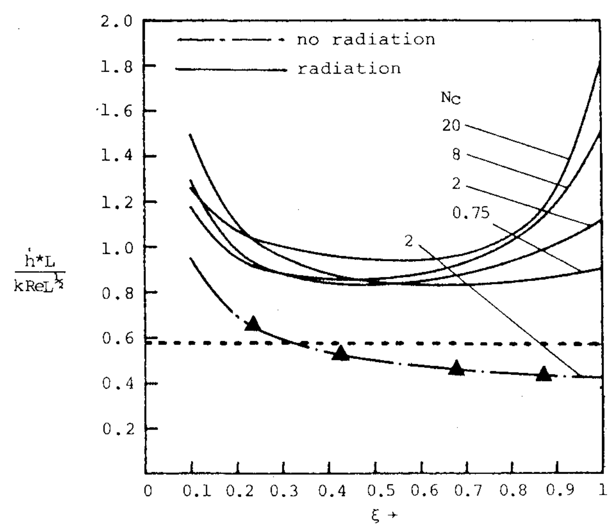

Fig. 2 The modified heat-transfer coefficients along the plate fin for $N=1$, $C_T=0.5$, $Pr=0.7$, and various values of Nc. ---, simple model; ▲, Sparrow.[4]

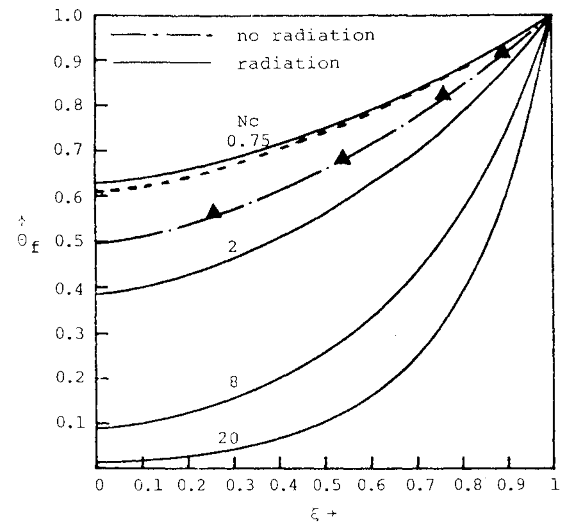

Fig. 3 The temperature distribution along the plate fin for $N=1$, $C_T=0.5$, and various values of Nc. ---, simple model; ▲, Sparrow.[4]

$$\frac{Q}{k(T_0-T_\infty)Re_L^{1/2}}=2\int_0^1\left(-\frac{\partial\theta}{\partial\eta}\right)$$

$$\times\left[1+\frac{4(\theta+C_T)^3}{3N}\right]\Big/\xi^{1/2}\mathrm{d}\xi,\ \text{at}\ \eta=0 \tag{15}$$

or

$$\frac{Q}{k(T_0-T_\infty)Re_L^{1/2}}=\frac{2}{N_c}\frac{\mathrm{d}\theta_f}{\mathrm{d}\xi}\bigg|_{\xi=1} \tag{16}$$

The results of the overall rate of heat transfer Q from the fin are shown in Fig. 1 for various values of N_c. Figure 1 indicates that the overall heat-transfer rate of the fin with radiative effect is higher than that of the fin without radiative effect. The agreement of the results from the simple model with those of Ref. 4, and the special case ($q^r=0$) in this paper is good.

The distributions of the modified local heat-transfer coefficient $\hat{h}^*$ for forced convection and radiation along the fin with different N_c are shown in Fig. 2. The modified heat transfer coefficient can be obtained from Eq. (14). For higher values of N_c, the fin is more nonisothermal. Although the local heat-transfer coefficient without radiative effects[4] monotonically decreases in the fluid flow direction, the modified local heat transfer coefficients computed in this paper with fixed radiative effect do not vary monotonically. In the direction from tip to base, those coefficients decrease at first, attain a minimum, and then increase. Figure 2 also shows that the local heat transfer coefficient with radiative effect is higher than that without radiative effect.

Figure 3 presents fin temperature distributions for forced convection flow with radiative effect. In this figure, it is shown that the fin temperature decreases monotonically from the root to tip. The figure also shows that the larger values of N_c give rise to larger fin temperature variations and the fin temperature without radiative effect is always higher than that with radiative effect.

Conclusion

The analysis in this Note has yielded the results of physical fin for forced convection flow with radiative effect under the optically thick limit approximation. The agreement of the results for special case $q^r=0$ with Ref. 4 is remarkable.

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Improved Free Convective Heat-Transfer Correlations in the Near-Critical Region

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Nomenclature

C_p = specific heat at constant pressure
$\bar{C}_p$ = mean-integral heat capacity, $\bar{C}_p=(i_w-i_\infty)/(T_w-T_\infty)$
D = diameter
g = gravitational constant
Gr = Grashof number, $(gD^3\rho^2/\mu^2)[(\rho_\infty-\rho_w)/\rho_w]$
h = heat-transfer coefficient
i = enthalpy
k = thermal conductivity
L = characteristic length
Nu = Nusselt number, hL/k
Pr = Prandtl number, $\mu C_p/k$
Ra = Rayleigh number, $Gr\cdot Pr$
T = temperature
μ = absolute viscosity
ρ = density

Subscripts

f = evaluated at the film temperature, $T_f=(T_w+T_\infty)/2$
w = evaluated at the wall temperature
∞ = evaluated at the freestream (bulk) temperature

Introduction

A VARIETY of correlations have been developed for prediction of free convective heat-transfer rates[1-7] (see Table 1). Discrepancies exist between these correlations when they are applied in the near-critical region. The most significant sources of discrepancy appear to be the influences of 1) the particular reference temperature used in the evaluation of the physical properties, 2) the particular physical properties selected in reducing the dimensional experimental data to the dimensionless variables, and 3) the differences in values of the physical properties used by the various investigators.

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